EFFECT MODIFICATION

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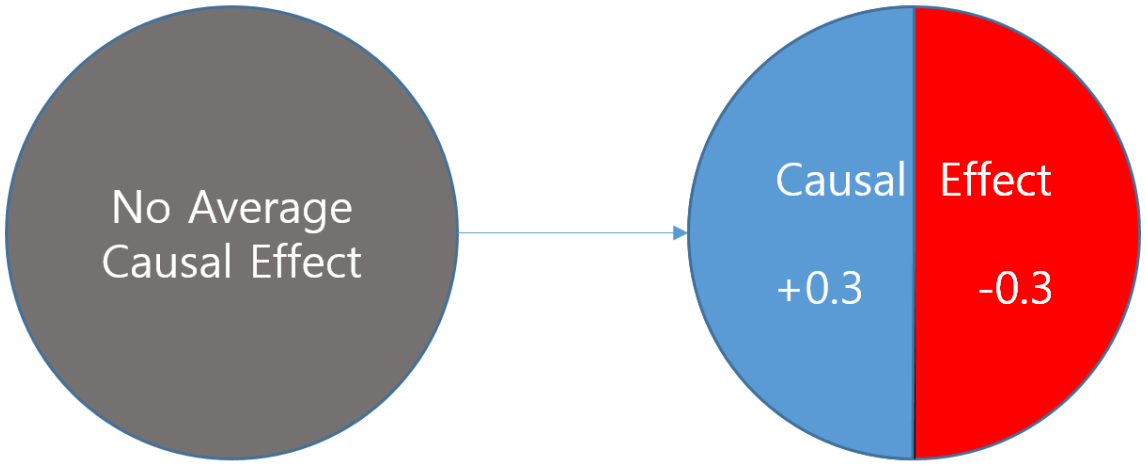
4. Effect Modification

Motivation :

Null average causal effect in the population does not imply a null average causal effect in a particular subset of the population.



There is not the causal effect of treatment.
Rather, the causal effect depends on the characteristics of the particular population under study.

4.1 Definition of effect modification

Table 4.1

	V	Y^0	Y^1
Rheia	1	0	1
Demeter	1	0	0
Hestia	1	0	0
Hera	1	0	0
Artemis	1	1	1
Leto	1	0	1
Athena	1	1	1
Aphrodite	1	0	1
Persephone	1	1	1
Hebe	1	1	0
Kronos	0	1	0
Hades	0	0	0
Poseidon	0	1	0
Zeus	0	0	1
Apollo	0	1	0
Ares	0	1	1
Hephaestus	0	0	1
Cyclope	0	0	1
Hermes	0	1	0
Dionysus	0	1	0

Average causal effect was null.

What is the average causal effect of A on Y in women? And in men?.

Let's restrict the analysis with $V = 1$(female).

$\Pr(Y^{a=1} = 1|V = 1) = 6/10 = 0.6,$
$\Pr(Y^{a=0} = 1|V = 1) = 4/10 = 0.4.$

Causal risk ratio : 1.5, Causal risk difference : 0.2

So, on average, heart transplant A increases the risk of death Y in women.

4.1 Definition of effect modification

V is a modifier of the effect of A on Y when the average causal effect of A on Y varies across levels of V.

The presence of effect modification depends on the effect measure being used. For example,

$$\Pr\left[Y^{a=0} = 1 \mid V = 1\right] = 0.8$$
$$\Pr\left[Y^{a=1} = 1 \mid V = 1\right] = 0.9$$
$$\Pr\left[Y^{a=0} = 1 \mid V = 0\right] = 0.1$$
$$\Pr\left[Y^{a=1} = 1 \mid V = 0\right] = 0.2$$

Multiplicative, but not additive, effect modification by V.

4.2 Stratification to identify effect modification

Table 2.2

	L	A	Y
Rheia	0	0	0
Kronos	0	0	1
Demeter	0	0	0
Hades	0	0	0
Hestia	0	1	0
Poseidon	0	1	0
Hera	0	1	0
Zeus	0	1	1
Artemis	1	0	1
Apollo	1	0	1
Leto	1	0	0
Ares	1	1	1
Athena	1	1	1
Hephaestus	1	1	1
Aphrodite	1	1	1
Cyclope	1	1	1
Persephone	1	1	1
Hermes	1	1	0
Hebe	1	1	0
Dionysus	1	1	0

Table 4.2
Stratum $V = 0$

	L	A	Y
Cybele	0	0	0
Saturn	0	0	1
Ceres	0	0	0
Pluto	0	0	0
Vesta	0	1	0
Neptune	0	1	0
Juno	0	1	1
Jupiter	0	1	1
Diana	1	0	0
Phoebus	1	0	1
Latona	1	0	0
Mars	1	1	1
Minerva	1	1	1
Vulcan	1	1	1
Venus	1	1	1
Seneca	1	1	1
Proserpina	1	1	1
Mercury	1	1	0
Juventas	1	1	0
Bacchus	1	1	0

20 Romans(V=0) and 20 Greeks(V=1).

$$\Pr(Y^{a=1} = 1 | V = 1) = 0.5,$$
$$\Pr(Y^{a=0} = 1 | V = 1) = 0.5$$
$$\Pr(Y^{a=1} = 1 | V = 0) = 0.6,$$
$$\Pr(Y^{a=0} = 1 | V = 0) = 0.3$$

It is possible that nationality is simply a marker for the causal factor.

Neutral term :
surrogate effect modifier

4.3 Why care about effect modification

There is not the causal effect of treatment.

Rather, the causal effect depends on the characteristics of the particular population under study.

The average causal effect in one population may not be transportable to other populations.

Need to check if there is an effect modification.

4.4 Stratification as a form of adjustment

Standardization (or IP weighting) is used to adjust for L.

Stratification is used to identify effect modification by V.

But the use of stratification as a method to adjust for L is also widespread.

4.5 Matching as another form of adjustment

The goal of matching :

To construct a subset of the population in which the variables L have the same distribution in both the treated and the untreated.

The matched population is a subset of the original study population. So the distribution of causal effect modifiers differ from that in the original.

4.6 Effect modification and adjustment methods

Standardization, IP weighting, stratification, matching are different approaches to estimate average causal effects, but they estimate different types of causal effects.

Let us compute the effect of heart transplant A on high blood pressure Z.

We assume that exchangeability $Z^a \perp\!\!\!\perp A|L$ and positivity hold.

4.6 Effect modification and adjustment methods

Table 4.3

	L	A	Z
Rheia	0	0	0
Kronos	0	0	1
Demeter	0	0	0
Hades	0	0	0
Hestia	0	1	0
Poseidon	0	1	0
Hera	0	1	1
Zeus	0	1	1
Artemis	1	0	1
Apollo	1	0	1
Leto	1	0	0
Ares	1	1	1
Athena	1	1	1
Hephaestus	1	1	1
Aphrodite	1	1	0
Cyclope	1	1	0
Persephone	1	1	0
Hermes	1	1	0
Hebe	1	1	0
Dionysus	1	1	0

Standardization and IP weighting :

We can calculate

$\Pr\left[Z^{a=1}=1\right] / \Pr\left[Z^{a=0}=1\right] = 0.8$

(Entire population)

Stratification :

If we adjust for the factor L to compute the effect of A on Z,

$\Pr\left[Z^{a=1}=1 \mid L=0\right] / \Pr\left[Z^{a=0}=1 \mid L=0\right] =$ 2.0 in the stratum $L=0$ and

$\Pr\left[Z^{a=1}=1 \mid L=1\right] / \Pr\left[Z^{a=0}=1 \mid L=1\right] =$ 0.5 in the stratum $L=1$

4.6 Effect modification and adjustment methods

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Persephone	1	1	0
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Hebe	1	1	0
Dionysus	1	1	0

Matching :

For individual in $(A = 0, L = l)$,
randomly select individual in $(A = 1, L = l)$.
If we calculate the effect in the untreated,
we matched Rheia-Hestia, Krnos-Poseidon,
Demeter-Hera, Hades-Zeus for $L = 0$,
and Artemis-Ares, Apollo-Aphrodite, Leto-Hermes
for $L = 1$.

$\Pr\left[Z^{a=1} = 1 \mid A = 0\right] / \Pr[Z = 1 \mid A = 0] = 1.0.$

Their discrepancy results from the different causal questions rather than from the choice of analytic approach.